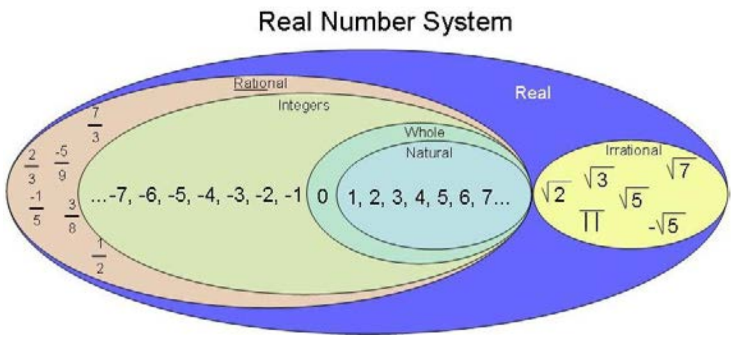
Algebra I



Day 1	Day 2	Day 3	Day 5	Day 5
Unit 2 Topic 7 Lesson 5 Factoring	Unit 2 Topic 7 Lesson 5 Factoring	Unit 2 Topic 7 Lesson 6 Factoring	Unit 2 Topic 7 Lesson 6 Factoring	Unit 2 Topic 7 Lesson 7 Factoring special Cases
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
Unit 2 Topic 7 Lesson 8 Factoring by Grouping	Unit 2 Topic 7 Lesson 8 Factoring by Grouping	Unit 2 Topic 7 Lesson 9-Simplifying Rational Expressions	Unit 2 Topic 7 Lesson 9-Simplifying Rational Expressions	Unit 2 Topic 7 Lesson 10-Simplifying Rational Expressions

Name ______________________________ Class ____________ Date ___________

7-5 Reteaching

If a trinomial of the form $x^2 + bx + c$ can be written as the product of two binomials, then:

- The coefficient of the x-term in the trinomial is the sum of the constants in the binomials.
- The trinomial's constant term is the product of the constants in the binomials.

What is the factored form of $x^2 + 12x + 32$?

To write the factored form, you are looking for two factors of 32 that have a sum of 12.

Solve Make a table showing the factors of 32.

Factors of 32	Sum of Factors
1 and 32	33
2 and 16	18
4 and 8	12

$x^2 + 12x + 32 = (x + 4)(x + 8)$

Check $(x + 4)(x + 8)$

$x^2 + 8x + 4x + 32$ Use FOIL Method.

$x^2 + 12x + 32$ Combine the like terms.

Solution: The factored form of $x^2 + 12x + 32$ is $(x + 4)(x + 8)$.

Exercises

Factor each expression.

1. $x^2 + 9x + 20$

2. $y^2 + 12y + 35$

3. $z^2 + 8z + 15$

4. $a^2 + 11a + 28$

5. $b^2 + 10b + 16$

6. $c^2 + 12c + 27$

7. $d^2 + 6d + 5$

8. $e^2 + 15e + 54$

9. $f^2 + 11f + 24$

Name ________________________________ Class ____________ Date __________

7-5 Reteaching (continued)

Some factorable trinomials in the form of $x^2 + bx + c$ will have negative coefficients. The rules for factoring are the same as when the x-term and the constant are positive.

- The coefficient of the x-term of the trinomial is the sum of the constants in the binomials.
- The trinomial's constant term is the product of the constants in the binomials.

However, one or both constants in the binomial factors will be negative.

What is the factored form of $x^2 - 3x - 40$?

To write the factored form, you are looking for two factors of –40 that have a sum of –3. The negative constant will have a greater absolute value than the positive constant.

Solve Make a table showing the factors of –40.

Factors of –40	Sum of Factors
1 and –40	–39
2 and –20	–18
4 and –10	–6
5 and –8	–3

$x^2 - 3x - 40 = (x - 8)(x + 5)$

Check

$(x - 8)(x + 5)$	
$x^2 + 5x - 8x - 40$	Use FOIL Method.
$x^2 + (-3x) - 40$	Combine the like terms.

Solution: The factored form of $x^2 - 3x - 40$ is $(x - 8)(x + 5)$.

Exercises

Factor each expression.

10. $s^2 + 2s - 35$ **11.** $t^2 - 4t - 32$ **12.** $u^2 + 6u - 27$

13. $v^2 - 2v + 48$ **14.** $w^2 - 8w - 9$ **15.** $x^2 + 3x - 18$

Name ____________________________________ Class _____________ Date ____________

7-6 Reteaching

You can use your knowledge of prime numbers to help you factor some trinomials as two binomials. A prime number has only 1 and itself as factors. For trinomials of the form $ax^2 + bx + c$, if a is a prime number then you already know the first term of each binomial: ax and $1x$. Then list the factors that will multiply to produce c. Use guess and check to find the factor pair that will add to b.

What is the factored form of $7x^2 + 31x + 12$?

$7x^2 + 31x + 12 = (7x \quad)(1x \quad)$ — a is 7, which is prime, so the factors are 7 and 1.

$= (7x \quad)(x \quad)$ — You don't need the 1 in front of the variable, so drop it.

$7x^2 + 31x + 12 = (7x + \quad)(x + \quad)$ — The trinomial has two plus signs, so the binomials also have plus signs.

Because c is 12, find factor pairs that multiply to 12: (1 and 12), (2 and 6), (3 and 4).

Try each pair in the expression to see if the INNER and OUTER products add to b, or 31.

$(7x + 1)(x + 12) = 7x^2 + x + 84x = 7x^2 + \mathbf{85x} + 12$ (NO)

$(7x + 2)(x + 6) = 7x^2 + 2x + 42x = 7x^2 + \mathbf{44x} + 12$ (NO)

$(7x + 3)(x + 4) = 7x^2 + 3x + 28x = 7x^2 + \mathbf{31x} + 12$ (YES)

The factored form of $7x^2 + 31x + 12$ is $(7x + 3)(x + 4)$.

Exercises

Factor each expression.

1. $3x^2 + 14x + 8$

2. $5y^2 + 43y + 24$

3. $2z^2 + 19z + 42$

4. $11a^2 + 39a + 18$

5. $13b^2 + 58b + 24$

6. $23c^2 + 56c + 20$

7. $7d^2 + d - 8$

8. $3e^2 + 20e - 32$

9. $19f^2 + 10f - 9$

10. $5s^2 - 18s + 16$

11. $17t^2 - 12t - 5$

12. $29u^2 + 48u - 20$

Name ______________________________ Class ___________ Date ___________

7-6 Reteaching (continued)

If you are given the area and one side of a rectangle, you can find the second side by factoring the trinomial. One binomial is the width and the other binomial is the length.

The area of a rectangular swimming pool is $6x^2 + 11x + 3$. The width of the pool is $2x + 3$. What is the length of the pool?

You are given the area and length of the pool. Set up an equation with what you are given and solve or length.

$6x^2 + 11x + 3 = (2x + 3)(\square\square\square)$	Area = length × width.
$6x^2 + 11x + 3 = (2x + 3)(3x\ \square\square)$	$6x^2 = (2x)(3x)$, so the first term of the second binomial is $3x$.
$6x^2 + 11x + 3 = (2x + 3)(3x + \square)$	The trinomial has two plus signs, so the sign for the second binomial must also be plus.
$6x^2 + 11x + 3 = (2x + 3)(3x + 1)$	The value of c is 3. Since 3 = 3 × 1, the second term must be 1.

Multiply to check your answer. Use FOIL.

$(2x + 3)(3x + 1) = 6x^2 + 2x + 9x + 3 = 6x^2 + 11x + 3$ ✓

The length of the swimming pool is $3x + 1$.

Exercises

13. The area of a rectangular cookie sheet is $8x^2 + 26x + 15$. The width of the cookie sheet is $2x + 5$. What is the length of the cookie sheet?

14. The area of a rectangular lobby floor in the new office building is $15x^2 + 47x + 28$. The length of one side of the lobby is $5x + 4$. What is the width?

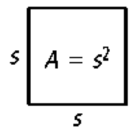
15. The area of a rectangular school banner is $12x^2 + 13x – 90$. The width of the banner is $3x + 10$. What is the length of the banner?

16. The distance a train has traveled is $6x^2 – 23x + 20$. The train's average speed is $3x – 4$. How long has the train been traveling?

Name ____________________________________ Class _____________ Date ____________

7-7 Reteaching

The area of a square is given by $A = s^2$, where s is a side length. When the side length is a binomial, the area can be written as a perfect-square trinomial. If you are given the area of such a square, you can use factoring to write an expression for a side length.



A mosaic is made of small square tiles called tesserae. Suppose the area of one tessera is $9x^2 + 12x + 4$. What is the length of one side of a tessera?

Because the tile is a square, you know the side lengths must be equal. Therefore, the binomial factors of the trinomial must be equal.

$x^2 + 12x + 4 = (\square\ \square\ \square)^2$	This is a perfect square trinomial and can be factored as the square of a binomial.
$9x^2 = (3x)^2$	$9x^2$ and 4 are perfect squares. Write them as squares.
$4 = 2^2$	
$2(3x)(2) = 12x$	Check that $12x$ is twice the product of the first and last terms. It is, so you are sure that you have a perfect-square trinomial.
$9x^2 + 12x + 4 = (3x + 2)^2$	Rewrite the equation as the square of a binomial.

Multiply to check your answer.

$$(3x + 2)(3x + 2) = 9x^2 + 6x + 6x + 4 = 9x^2 + 12x + 4 \checkmark$$

The length of one side of the square is $3x + 2$.

Exercises

Factor each expression to find the side length.

1. The area of a square oil painting is $4x^2 + 28x + 49$. What is the length of one side of the painting?

2. You are installing linoleum squares in your kitchen. The area of each linoleum square is $16x^2 - 24x + 9$. What is the length of one side of a linoleum square?

3. You are building a table with a circular top. The area of the tabletop is $(25x^2 - 40x + 16)\pi$. What is the radius of the tabletop?

4. A fabric designer is making a checked pattern. Each square in the pattern has an area of $x^2 - 16x + 64$. What is the length of one side of a check?

Name ____________________ Class __________ Date __________

7-7 Reteaching (continued)

Some binomials are a difference of two squares. To factor these expressions, write the factors so the *x*-terms cancel and you are left with two perfect squares.

What is the factored form of $4x^2 - 9$?

$4x^2 - 9 = (\square + \square)(\square - \square)$	Both $4x^2$ and 9 are perfect squares. You know the signs of the factors will be opposite, so the *x*-terms will cancel out.
$\sqrt{4x^2} = 2x$ $\sqrt{9} = 3$	Find the square root of each term.
$(2x + 3)(2x - 3)$	Write each term as a binomial with opposite signs, so the *x*-terms will cancel out.

Multiply to check your answer.

$$(2x + 3)(2x - 3) = 4x^2 + 6x - 6x - 9$$
$$= 4x^2 - 9 \checkmark$$

The factored form of $4x^2 - 9$ is $(2x + 3)(2x - 3)$.

Exercises

Factor each expression.

5. $9x^2 - 4$ **6.** $25x^2 - 49$ **7.** $144x^2 - 1$

8. $64x^2 - 25$ **9.** $49x^2 - 16$ **10.** $36x^2 - 49$

11. $81x^2 - 16$ **12.** $16x^2 - 121$ **13.** $25x^2 - 144$

14. $16x^2 - 9$ **15.** $x^2 - 81$ **16.** $4x^2 - 49$

Name ____________________________ Class ____________ Date ____________

7-8 Reteaching

You can factor some higher-degree polynomials by grouping terms and factoring out the GCF to find the common binomial factor. Make sure to factor out a common GCF from all terms first before grouping.

What is the factored form of $2b^4 - 8b^3 + 10b^2 - 40b$?

$2b^4 - 8b^3 + 10b^2 - 40b = 2b(b^3 - 4b^2 + 5b - 20)$	$2b$ is the GCF of all four terms. Factor out $2b$ from each term.
$= 2b[b^2(b - 4) + 5(b - 4)]$	Group terms into pairs and look for the GCF of each pair. b^2 is the GCF of the first pair, and 5 is the GCF of the second pair.
$= 2b(b^2 + 5)(b - 4)$	$b - 4$ is the common binomial factor. Use the Distributive Property to rewrite the expression.

Multiply to check your answer.

$2b(b^2 + 5)(b - 4) = 2b(b^3 + 5b - 4b^2 - 20)$	Multiply $b^2 + 5$ and $b - 4$.
$= 2b^4 + 10b^2 - 8b^3 - 40b$	Multiply by $2b$.
$= 2b^4 - 8b^3 + 10b^2 - 40b$ ✓	Reorder the terms by degree.

The factored form of $2b^4 - 8b^3 + 10b^2 - 40b$ is $2b(b^2 + 5)(b - 4)$.

Exercises

Factor completely. Show your steps.

1. $4x^4 + 8x^3 + 12x^2 + 24x$

2. $24y^4 + 6y^3 + 36y^2 + 9y$

3. $72z^4 + 48z^3 + 126z^2 + 84z$

4. $2e^4 - 8e^3 + 18e^2 - 72e$

5. $12f^3 - 36f^2 + 60f - 180$

6. $16g^4 - 56g^3 + 64g^2 - 224g$

7. $56m^3 - 28m^2 - 42m + 21$

8. $40n^4 - 60n^3 - 50n^2 + 75n$

9. $60x^3 - 90x^2 - 30x + 45$

10. $12p^5 + 8p^4 + 18p^3 + 12p^2$

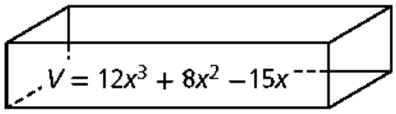
11. $6r^3 + 9r^2 - 60r$

12. $20s^6 - 50s^5 - 30s^4$

Name ______________________ Class ____________ Date ____________

7-8 Reteaching (continued)

Polynomials can be used to express the volume of a rectangular prism. They can sometimes be factored into 3 expressions to represent possible dimensions of the prism. The three factors are the length, width, and height.



The plastic storage container to the right has a volume of $12x^3 + 8x^2 - 15x$. What linear expressions could represent possible dimensions of the storage container?

$12x^3 + 8x^2 - 15x = x(12x^2 + 8x - 15)$	Factor out x, the GCF for all three terms.
$= x(12x^2 + 18x - 10x - 15)$	ac is –180 and b is 8. Break $8x$ into two terms that have a sum of $8x$ and a product of $-180x^2$.
$= x[6x(2x + 3) - 5(2x + 3)]$	Group the terms into pairs and factor out the GCF from each pair. The GCF of the first pair is $6x$. The GCF of the second pair is –5.
$= x(6x - 5)(2x + 3)$	$2x + 3$ is the common binomial term. Use the Distributive Property to reorganize the factors.

Multiply to check your answer.

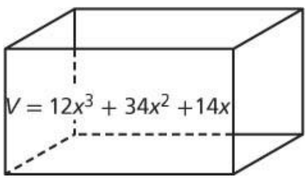
$x(6x - 5)(2x + 3) = x(12x^2 + 18x - 10x - 15)$	Multiply $6x - 5$ and $2x + 3$.
$= x(12x^2 + 8x - 15)$	Combine like terms.
$= 12x^3 + 8x^2 - 15x$ ✓	Multiply by x.

Possible dimensions of the storage container are x, $6x - 5$, and $2x + 3$.

Exercises

Find linear expressions for the possible dimensions of each rectangular prism.

13.



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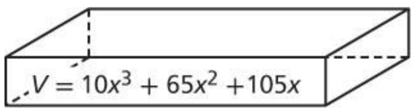

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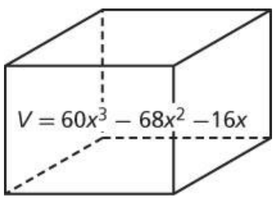

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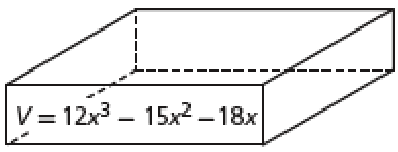

Name____________________________________ Class _____________ Date____________

7-9 Reteaching

Rational expressions may be in the form of *monomials* or *polynomials.*

Simplifying rational expressions is similar to simplifying numerical fractions where common factors are taken out.

Example: $\frac{x-2}{3x-6} = \frac{x-2}{3(x-2)} = \frac{1}{3}$

Excluded values are those that make the denominator 0. A denominator can not equal 0, so these values are not part of the solution. Consider not only the solution, but also the origin expression to figure out the excluded values.

Problem

What is the simplified form of $\frac{2a^3}{4a^2}$? State any excluded values.

Solve Monomials: reduce numbers; cancel out like variables

$$\frac{2a^3}{4a^2} = \frac{2 \cdot a \cdot a \cdot a}{2 \cdot 2 \cdot a \cdot a} = \frac{a}{2}$$

The simplified form is $\frac{a}{2}$ when $a \neq 0$.

What is the simplified form of $\frac{x^2+4x+4}{x+2}$? State any excluded values.

Solve Polynomials: cancel out factors or groups of factors

$$\frac{x^2+4x+4}{x+2} = \frac{(x+2)(x+2)}{x+2} = x+2$$

The simplified form is $x + 2$ when $x \neq -2$.

Recognizing Opposite Factors

You can find the opposite of a number by multiplying by –1. For example, the opposite of 3 is (–1)(3) = –3.

Similarly, multiplying a polynomial by –1 results in its opposite. For example, the opposite of $x - 2$ is $(-1)(x - 2) = -x + 2$. It can also be written as $2 - x$.

Problem

Write the opposite of $(20 - x)$ two ways.

Solve Multiply by (-1) to find the opposite.

$(-1)(20 - x) = -20 + x$ or $x - 20$

Name ______________________ Class ____________ Date ____________

7-9 Reteaching (continued)

Exercises

Simplify each expression. State any excluded values.

1. $\frac{a^2}{a}$

2. $\frac{4x^3}{16x^2}$

3. $\frac{cd^2}{3c^2d}$

4. $\frac{lm}{l^2m^2n}$

5. $\frac{64y}{16y^2x}$

6. $\frac{2x^2-4x}{x}$

7. $\frac{5x^3-15x^2}{x-3}$

8. $\frac{x^2+5x+6}{x+3}$

9. $\frac{2b+4}{4}$

10. $\frac{3a+15}{15}$

11. $\frac{3p-21}{18}$

12. $\frac{4}{4y-8}$

13. $\frac{7z-28}{14z}$

14. $\frac{9}{18-81a}$

15. $\frac{5}{35-5c}$

16. $\frac{2q+2}{q^2+4q+3}$

17. $\frac{a+2}{a^2+4a+4}$

18. $\frac{2x-2}{2-2x}$

19. $\frac{9-x^2}{x-3}$

20. $\frac{2a+4}{2}$

Write the opposite expression and simplify the opposite expression.

21. $\frac{10b^5}{40b^4}$

22. $\frac{36-z^2}{4z-24}$

23. $\frac{x^2-16}{x-4}$

24. $\frac{30+2z}{14+4z}$

Name____________________________________ Class_____________ Date ____________

7-10 Reteaching

A few important rules are needed for successful division of a polynomial. When dividing by a monomial, a single term, remember to divide each polynomial term by the monomial and reduce the fraction.

Problem

What is $(8x^3 - 3x^2 + 16x) \div 2x^2$?

Solve

$(8x^3 - 3x^2 + 16x) \div 2x^2$ — Division equals multiplication by the reciprocal.

$= (8x^3 - 3x^2 + 16x) \bullet \dfrac{1}{2x^2}$ — Multiply by $\dfrac{1}{2x^2}$, the reciprocal of $2x^2$.

$= \dfrac{8x^3}{2x^2} - \dfrac{3x^2}{2x^2} + \dfrac{16x}{2x^2}$ — Use the Distributive Property.

$= 4x^1 - \dfrac{3}{2}x^0 + \dfrac{8}{x}$ — Subtract exponents when dividing powers with the same base.

$= 4x - \dfrac{3}{2} + \dfrac{8}{x}$ — Simplify.

Exercises

Divide.

1. $(2x^2 - 9x + 18) \div 2x$

2. $(16x^4 - 64) \div 4x^3$

3. $(x^5 - 3x^4 + 10x^3 - \frac{3}{4}x^2 - 6) \div 3x^2$

4. $(5x^3 - 25x^2 - 1) \div 5x$

When a polynomial (many terms) is divided by a binomial (2 terms), the polynomial terms should be in order from highest to lowest exponent.

To make the polynomial $-4 + 2x - 16x^2 + 3x^3$ division ready, put it in the correct order for division, from greatest exponent to lowest. The correct order for $-4 + 2x - 16x^2 + 3x^3$ to be division ready is $3x^3 - 16x^2 + 2x - 4$.

For any gaps or missing exponents, the place is held with 0. For example, $x^2 + 1$ becomes $x^2 + 0x + 1$, $0x$ being the placeholder for the x term.

7-10 Reteaching (continued)

Dividing a polynomial by a binomial is similar to long division.

Problem

What is $(p^2 - 3p + 2) \div (p - 2)$?

Solve

$$p-2\overline{)p^2-3p+2}$$

Write the problem as long division.

$$\begin{array}{r} p \\ p-2\overline{)p^2-3p+2} \end{array}$$

Ask how many times p goes into p^2 (p times). The variables must align by exponent, so the p goes above 23p since both match.

$$\begin{array}{r} p \\ p-2\overline{)p^2-3p+2} \\ \underline{p^2-2p} \\ -1p+2 \end{array}$$

Multiply p times $(p - 2)$. Subtract the product p^2 2 2p. Bring down 2.

$$\begin{array}{r} p-1 \\ p-2\overline{)p^2-3p+2} \\ \underline{p^2-2p} \\ -1p+2 \\ \underline{-1p+2} \end{array}$$

Determine how many times p goes into $-1p$ (-1 times). Multiply (-1) times $(p - 2)$ to get $-1p + 2$. Subtract the product $-1p + 2$. There is no remainder.

So $p - 2$ goes into $p^2 - 3p + 2$ exactly $p - 1$ times with no remainder.

Exercises

Divide.

5. $(d^2 + 4d - 12) \div (d - 2)$

6. $(y^2 - 4y + 4) \div (y - 2)$

7. $(x^2 - 2x + 1) \div (x - 1)$

8. $(b^2 - b - 20) \div (b - 5)$